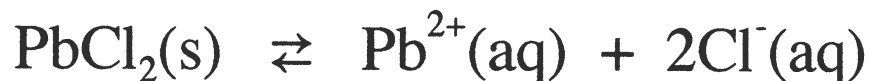


## The Solubility Product

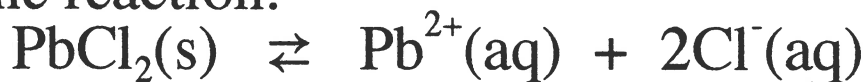
- when substances that are only slightly soluble dissolve in water, very little is required to form a saturated solution
  - > a heterogeneous equilibrium is created between the solid and its ions



- a saturated solution is said to be a **dynamic equilibrium** because the rate of dissolving equals the rate of precipitation and both reactions continue to occur even though there are no macroscopic changes
- saturated solutions best represented with a **double arrow** in their dissociation equations

- an equilibrium expression can be written for a saturated solution
  - > **SOLUBILITY PRODUCT EXPRESSION**

for the reaction:



$$K_{\text{sp}} = [\text{Pb}^{2+}][\text{Cl}^{-}]^2$$

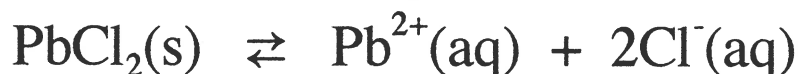
$K_{\text{sp}}$  = **solubility product constant**

- $K_{\text{sp}}$  is referred to as an **ion product**
  - > it has no units and no denominator because the reactant is a solid
- large  $K_{\text{sp}}$  values indicate the products are favoured
  - > the **larger the  $K_{\text{sp}}$** , the greater the number of ions and the greater the solubility of the compound

ex. Write the  $K_{\text{sp}}$  expression for the following equilibria:



- consider the following saturated solution:



- the equilibrium can be reached from either the reactants or the products
- if reached from the reactants,  $\text{PbCl}_2(\text{s})$  is dissolved in water and

$$[\text{Pb}^{2+}] = [\text{PbCl}_2(\text{aq})]$$

$$[\text{Cl}^{-}] = 2 \times [\text{PbCl}_2(\text{aq})]$$

$$[\text{Cl}^{-}] = 2 \times [\text{Pb}^{2+}]$$

- if the equilibrium is reached from the products, the above proportions are NOT necessarily true and the  $[\text{Cl}^{-}]$  and  $[\text{Pb}^{2+}]$  will depend on the solutions that are mixed together

Q. What are the  $[\text{Cl}^{-}]$  and  $[\text{Pb}^{2+}]$  when 25.0 mL of 0.025 M  $\text{Pb}(\text{NO}_3)_2(\text{aq})$  is mixed with 30.0 mL of 0.010 M  $\text{AlCl}_3(\text{aq})$ ?

$$[\text{Pb}(\text{NO}_3)_2] = \frac{25.0 \text{ mL} (0.025 \text{ M})}{55.0 \text{ mL}} = 0.011 \text{ M}$$

$$[\text{AlCl}_3] = \frac{30.0 \text{ mL} (0.010 \text{ M})}{55.0 \text{ mL}} = 5.45 \times 10^{-3} \text{ M}$$

$$[\text{Pb}^{2+}] = 0.011 \text{ M}$$

$$[\text{Cl}^{-}] = 3(5.5 \times 10^{-3} \text{ M}) = 0.016 \text{ M}$$

Example: Calculating  $K_{sp}$  from solubility data

100 mL of a saturated  $PbI_2$  solution was found to contain  $5.23 \times 10^{-2}$  g  $PbI_2(s)$ . Calculate  $K_{sp}$ .

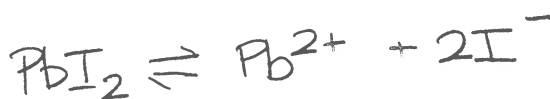
$$[PbI_2] = \frac{\text{mol}}{\text{L}} = (5.23 \times 10^{-2} \text{g}) \left( \frac{\text{mol}}{461.0 \text{g}} \right) \left( \frac{1}{0.100 \text{L}} \right)$$

$$= 1.13 \times 10^{-3} \text{M}$$

$$Pb = 207.2$$

$$I = 2(126.9)$$

$$\frac{461.0 \text{g}}{\text{mol}}$$



$$1.13 \times 10^{-3}$$

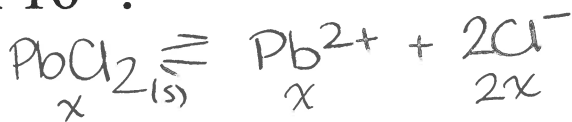
$$2(1.13 \times 10^{-3} \text{M})$$

$$= 2.27 \times 10^{-3} \text{M}$$

$$K_{sp} = [Pb^{2+}][I^{-}]^2$$
$$= (1.13 \times 10^{-3})(2.27 \times 10^{-3})^2$$
$$= 5.84 \times 10^{-9}$$

Example: Calculating solubility from  $K_{sp}$

What is the molar solubility of  $PbCl_2$  if the  $K_{sp}$  is  $1.8 \times 10^{-4}$ ?



$$K_{sp} = [Pb^{2+}][Cl^{-}]^2$$

$$K_{sp} = (x)(2x)^2$$

$$K_{sp} = 4x^3 = 1.8 \times 10^{-4}$$

$$x^3 = \frac{1.8 \times 10^{-4}}{4}$$

$$x^3 = 4.5 \times 10^{-5}$$

$$x = \sqrt[3]{4.5 \times 10^{-5}}$$

$$x = 0.035568933$$

molar solubility  
of  $PbCl_2$

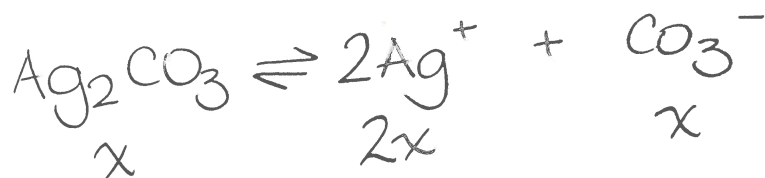
$$= 0.036 \frac{\text{mol}}{\text{L}}$$

Example: Calculating ion concentrations from  $K_{sp}$

What is the  $[Ag^+]$  in a saturated solution of  $Ag_2CO_3$ ?

\*look up  $K_{sp}$  in data booklet

$$K_{sp} Ag_2CO_3 = 8.5 \times 10^{-12}$$



$$K_{sp} = [Ag^+]^2 [CO_3^{2-}]$$

$$K_{sp} = (2x)^2 x$$

$$K_{sp} = 4x^3 = 8.5 \times 10^{-12}$$

$$x = \sqrt[3]{\frac{8.5 \times 10^{-12}}{4}}$$

$$x = 1.2856 \times 10^{-4}$$

$$[Ag^+] = 2x = 2(1.2856 \times 10^{-4})$$

$$[Ag^+] = 2.6 \times 10^{-4} M$$

Note: In Chem12, there are only two types of salts that need to be considered: AB and AB<sub>2</sub> ~~A<sub>2</sub>B~~

- if x is the solubility of a salt, then the following relationships exist between K<sub>sp</sub> and the solubility of the salt

$$\text{(AB salt)} \quad K_{sp} = x^2 \quad \text{and} \quad x = \sqrt{K_{sp}}$$

$$\text{(AB}_2 \text{ or A}_2\text{B salt)} \quad K_{sp} = 4x^3 \quad \text{and} \quad x = \sqrt[3]{\frac{K_{sp}}{4}}$$

ex. The molar solubility of Ag<sub>2</sub>S is 1.3 x 10<sup>-17</sup> M. What is the K<sub>sp</sub> for Ag<sub>2</sub>S?

$$\begin{aligned} K_{sp} &= 4x^3 = 4(1.3 \times 10^{-17})^3 \\ &= 8.8 \times 10^{-51} \end{aligned}$$

ex. The value of K<sub>sp</sub> for AgCl is 1.8 x 10<sup>-10</sup>. What is the molar solubility of AgCl?

$$\begin{aligned} K_{sp} &= x^2 = 1.8 \times 10^{-10} \\ x &= \sqrt{1.8 \times 10^{-10}} \end{aligned}$$

$$[\text{AgCl}] = 1.3 \times 10^{-5} \text{ M}$$